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Is supernovae data in favour of isotropic cosmologies?

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Abstract

Most of the observational claims in cosmology are based on the assumption that the universe is isotropic and homogeneous so they essentially test different types of Friedmann models. This also refers to recent observations of supernovae Ia, which, within the framework of Friedmann cosmologies give strong support to negative pressure matter and also weaken the age conflict. In this essay we drop the assumption of homogeneity, though temporarily leaving the assumption of isotropy with respect to one point, and show that supernovae data can be consistent with a model of the universe with inhomogeneous pressure known as the Stephani model. Being consistent with supernovae data we are able to get the age of the universe in this model to be about 3.8 Gyr more than in its Friedmann counterpart.

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The standard isotropic cosmological models have intensively been studied as the models of the large-scale structure of the universe. One of the main reasons is their mathematical simplicity expressed in terms of the Cosmological Principle. There is of course some 'evidence' for these models to be the right ones from many different astronomical tests and especially from low-redshift linear Hubble expansion law (e.g. [1]). However, the situation is not so clear for large-redshift objects since the generalized Hubble law – the redshift-magnitude relation – becomes nonlinear and the effects of spatial curvature of the universe are important. In the past the main problem was that the luminosity function of prospective 'standard candles' (whose absolute magnitude is presumably known) was poorly known for most of them at redshifts $z \approx 1$. This is not the case for supernovae type Ia (SnIa) and these objects have recently been used to determine the curvature and consequently the matter content of the universe [2, 3]. The results of these investigations give strong support to Friedmann models with *negative pressure* matter such as the cosmological constant, domain walls or cosmic strings [4, 5]. It is a very strong claim, since, despite a very long story of the cosmological constant, [6] and a relatively long story of topological defects [7], people hardly believed in their large contribution to the total energy density of matter in the universe at the present epoch of the evolution.

In this essay we try to make an alternative proposal for the explanation of supernovae data and suggest an inhomogeneous model of the universe which belongs to the class of models known as the Stephani universes [8, 9]. We basically try to fit this model to SnIa data as given in [2]. Our model is described by the following metric tensor [10]

$$ds^2 = -\frac{c^2}{V^2}d\tau^2 + \frac{R^2}{V^2}\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right], \tag{1}$$

with

$$\begin{aligned} R(\tau) &= a\tau^2 + \tau, & (2)\\ V(\tau, r) &= 1 - \frac{a}{c^2}\left(a\tau^2 + \tau\right)r^2 & (3)\\ k(\tau) &= -4\frac{a^2}{c^2}R(\tau), & (4) \end{aligned}$$

where τ is the cosmic time coordinate and r is the radial coordinate. In Eq. (1) $R(\tau)$ is the generalized scale factor and $k(\tau)$ is the time-dependent spatial curvature index, so the spatial curvature of the universe may change during the evolution which is impossible in Friedmann models. The constant c is the velocity of light and the parameter a is measured in $\mathrm{km}^2\,\mathrm{s}^{-2}\,\mathrm{Mpc}^{-1}$. The physical meaning of a is that it measures non-uniformity of pressure (acceleration) of the model (see [10]). One is able to obtain the flat Friedmann model from (1) is one takes the limit $a \to 0$.

In the model described by the metric (1) the energy density ϱ depends on the cosmic time, similarly as in Friedmann models, but the pressure, p, is the function of both the time and radial coordinates [15]. This justifies its name as 'inhomogeneous pressure universe'. It is spherically symmetric and it can certainly be used as the first step towards the observational verification of inhomogeneous cosmologies. A general class of Stephani models is really inhomogeneous which means there are no Killing vectors in spacetime. The assumption of spherical symmetry is of course equivalent to dropping the assumption of the Cosmological Principle, provided we put an observer outside of the center of symmetry. In this essay we do not consider off-centre observers although the suitable relations are known [10]. Despite that, we can have an important effect on the observational relations at the center, because the light reaching the observer there was emitted from the off-center galaxies.

The spherically symmetric Stephani model is the model of concentric pressure spheres (pressure varies from sphere to sphere) and it can be put in some opposition to the Tolman model [11, 12] which is the model of concentric density spheres (energy density varies from sphere to sphere). Both models do not necessarily have to be used as models of the global geometry of the universe, but can also be applied to model local inhomogeneity (or void) in the universe. Kinematically, both models expand and in the Tolman model there is shear while in the Stephani model there is acceleration. Acceleration is the result of the combined effect of gravitational and inertial forces on the fluid which are unable to be separated and appears due to the spatial pressure gradient on the concentric spheres – the particles are accelerated in the direction from high-pressure regions to low-pressure regions.

We consider our investigations of Stephani models in the context of supernovae data as an important step towards understanding the large-scale structure of the universe because very few, if any, attempts to compare inhomogeneous models of the universe (see [13]) with astronomical data have been done so far. It was done for the first time by using a preliminary SnIa data [14] in [15] and in this essay we try to give new insight into the problem using large sample data given in [2].

The parameter space of Friedmann models contains of three parameters: the Hubble constant H_0, the deceleration parameter q_0 and the density of nonrelativistic matter Ω_{m0} reducing to just two of them in a flat universe.

The Stephani model under consideration is a simple generalization of a flat Friedmann model and its parameter space can mimic (as far as the redshift-magnitude relation is concerned) that of Friedmann with an important admission of the effect of pressure gradient (acceleration) in the universe.

The standard cosmological test – a redshift-magnitude relation – to second order in redshift z for Friedmann models reads as (e.g. [16])

$$\begin{aligned} m_{\rm B} &= M_{\rm B} + 25 + 5\log_{10} cz - 5\log_{10} H_0 \\ &+ 1.086\,(1-q_0)\,z + 0.2715\left[3(1+q_0)^2 - 4(1+\Omega_{m0})\right] z^2 + O(z^3), \end{aligned} \tag{5}$$

where m_B is the apparent bolometric magnitude of a galaxy, M_B is its absolute magnitude and z is the redshift. For Friedmann cosmologies the following relations between the parameters are fulfilled

$$\Omega_\Lambda = \frac{\Lambda}{3H_0^2} = \frac{1}{2}\Omega_{m0} - q_0, \qquad \frac{k}{H_0^2 R_0^2} = \frac{3}{2}\Omega_{m0} - q_0 - 1, \tag{6}$$

where Ω_Λ is the density of cosmological constant Λ, k the curvature index and R_0 the present value of the scale factor. The relation (5) was tested by supernovae data [2] and the best fit values of the cosmological parameters in flat ($k = 0 = \Omega_{m0} + \Omega_\Lambda - 1$) universe were claimed to be

$$q_0 = -0.55, \tag{7}$$

$$\Omega_{m0} = 0.3, \tag{8}$$

$$\Omega_\Lambda = 0.7, \tag{9}$$

for the Hubble'a constant

$$H_0 = 63 \ \text{kms}^{-1}\text{Mpc}^{-1}, \tag{10}$$

giving the best-fit age of the universe

$$t_0 = 14.9 \ \text{Gyr}. \tag{11}$$

.

The redshift-magnitude relation for Stephani universes has been found in [10]. Two exact cases were presented and the theoretical relations were plotted for a range of different parameter values. The relations were obtained following the method of Kristian & Sachs [17] of expanding all relativistic quantities in power series and truncating at a suitable order, though, one can use an exact relation too [18]. An analogous to (5) relation for the Stephani universe (1), to second order in redshift z, reads as [10, 15]

$$\begin{aligned} m_{\mathrm{B}} &= M_{\mathrm{B}} + 25 + 5\log_{10} cz - 5\log_{10}\tilde{H}_0 + 1.086\,(1-\tilde{q}_0)\,z \\ &+ 0.2715\left[3(1+\tilde{q}_0)^2 - 4(1+\tilde{\Omega}_{m0})\right]z^2 + O(z^3), \end{aligned} \tag{12}$$

where

$$\tilde{H}_0 = \frac{2a\tau_0 + 1}{a\tau_0^2 + \tau_0}, \tag{13}$$

$$\tilde{q}_0 = -4a\frac{a\tau_0^2 + \tau_0}{(2a\tau_0 + 1)^2}. \tag{14}$$

Equation (12) now takes the same functional form as equation (5), as was similarly pointed out in [10] with $\tilde{H}_0$ and $\tilde{q}_0$ replacing H_0 and q_0. We can think of $\tilde{H}_0$ and $\tilde{q}_0$ as a generalised Hubble parameter and deceleration parameter which are related to the age of the universe in a different way from the Friedmann case. The key question of interest here is therefore whether one can construct generalised parameters, $\tilde{H}_0$ and $\tilde{q}_0$, which are in good agreement with the supernovae data [2] but which correspond to a value of τ_0 which exceeds that Friedmann age with $H_0 = \tilde{H}_0$ and $q_0 = \tilde{q}_0$. More precisely, both relations (5) and (12) are equal provided a generalized density parameter $\tilde{\Omega}_{m0} = (1/3)(1+\tilde{q}_0)$. Assuming the following values for the replaced parameters

$$\tilde{H}_0 = 63 \ \mathrm{kms}^{-1}\mathrm{Mpc}^{-1}, \tag{15}$$

$$\tilde{q}_0 = -0.55, \tag{16}$$

we obtain the age of the universe in the Stephani model (1) to be

$$\tau_0 = 18.67 \quad \mathrm{Gyr}, \tag{17}$$

and $\tilde{\Omega}_{m0} = 0.15$. Then we have an agreement with supernovae data, provided the non-uniform pressure parameter is equal to

$$\mathrm{a} = 12.3 \ \mathrm{km^2 s^{-2} Mpc^{-1}}, \tag{18}$$

which translates into the value of the acceleration scalar [10] to be

$$\dot{u} = -2\frac{a}{c^2} r = -2.73 \cdot 10^{-10} r \mathrm{Mpc}^{-1}, \tag{19}$$

with r being the radial coordinate of the model. Since the non-uniform pressure parameter a is positive, then the high pressure region is at $r = 0$, while the low (negative) pressure regions are outside the center, so the particles are *accelerated away from the center*. This is similar effect as that caused by the positive cosmological constant $\Lambda > 0$ in Friedmann models, although the physical mechanism is somewhat different. However, in both cases it is worth to appeal to ideas from the theory of elementary particles [7] and especially to the notion of the energy of the vacuum. While in Friedmann models vacuum gives *constant pressure* on every spatial section of constant time, in Stephani models it gives the pressure which *depends on spatial coordinates*.

Finally, we emphasize that the result obtained here is based only on the studies of the redshift-magnitude relation. The value of the non-uniform pressure parameter in (18) should also be tested by the level of the microwave background anisotropies and other astronomical data.

Spherically symmetric models in which cosmic acceleration is also explained by the inhomogeneity in pressure (though except acceleration admitting shear) have been considered recently by Pascual-Sánchez [19]. They form a larger class than Stephani models.

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References

[1] Strauss M. A., Willick J. A., 1995, Phys. Rep., 261, 271

[2] S. Perlmutter et al., 1998, Astroph. Journ., to appear, and e-Print: astro-ph/9812133 (see also http://www-supernovae.lbl.gov/)

[3] Riess G. et al., 1998, Astron. Journ., 116, 1009

[4] Vilenkin A., 1985, Phys. Rep., 121, 265

[5] Garnavich et al., 1998, Astroph. Journ., 509, 74

[6] Einstein A., Sitzber. Preuss. Akad. Wiss., 1917, 1, 142

[7] Vilenkin, A., Shellard P., 'Cosmic Strings and Other Topological Defects' (CUP, Cambridge, 1994)

[8] H. Stephani, 1967, Commun. Math. Phys., 4, 137

[9] Dąbrowski M. P., 1993, Journ. Math. Phys., 34, 1447

[10] Dąbrowski M. P., 1995, Astroph. Journ., 447, 43

[11] Tolman R. C., 1934, Proc. Nat. Acad. Sci., 20, 169

[12] Bonnor W. B., 1974, MNRAS, 167, 55

[13] Krasiński A., 'Inhomogeneous Cosmological Models', (CUP, Cambridge, 1997)

[14] Perlmutter S. et al., 1997, Astroph. Journ., 483, 565

[15] Dąbrowski M. P., Hendry M. A., 1998, Astroph. Journ., 498, 67

[16] Dąbrowski M. P., Stelmach J. 1989, Astron. Journ., 97, 978

[17] Kristian J., Sachs R. K., 1966, Astroph. Journ., 143, 379

[18] Barrett R., Clarkson C., unpublished

[19] J.-F. Pascual-Sánchez, 'Cosmic Acceleration: Inhomogeneity Versus Vacuum Energy', e-Print: gr-qc/9905063